# Actuator disk wake model in RaNS

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### Abstract

A Wind Turbine (WT) wake model was integrated into a CFD code, in an attempt to bring the complex flow solving capabilities of state-of-the-art CFD solvers to the engineering level of wind farm development tools. For that purpose the tool does not require input other than standard manufacturer data on WTs (dimensions and power/thrust curves) and adds little computational cost to that already required by the CFD solver by itself. The present results are encouraging, as the model is able to predict satisfyingly the operating point (in the power/thrust curves) of the WT, thus estimating its performance, both in undisturbed flow and in the wake of another WT.

## Introduction

In off-shore applications, the present engineering solutions to estimate wake losses may prove unreliable due to the interaction between the numerous Wind Turbine (WT) wakes or less-than-ideal wind conditions, like heterogeneous or unsteady inflow. By modelling the WT presence into the iterating equations of a RaNS solver, the risk of increased error due to biased calibration is reduced. This solution should be of interest both at the layout planning and energy yield prediction phases.

## Objectives

An engineering minded RaNS-solved WT wake model should:

- Avoid need for precursor solutions to evaluate WT performance;
- Require minimum user input and calibration;
- Accurately estimate isolated WT wake behaviour;
- Predict wake interaction and WT behaviour in wake-disturbed inflow;

## Proposed model

An actuator disk model based on Froude's Actuator Disk and momentum theory model was integrated into a RaNS code, allowing WT performance to be estimated in real time as the CFD code's solution converges. This was possible by introducing sink terms into the momentum equations, equivalent to the effect the WT has on the flow passing through its rotor.

By using Betz's conclusions regarding one dimensional momentum theory applied to the actuator disk (2), the model can estimate the force-per-unit-area (using Equation 1) applied by the rotor on the flow and re-distribute it over the actuator disk surface, which represents the span of the WT rotor.

$$d\mathrm{F} = \frac{1}{2}\rho \mathrm{C_T} \mathrm{U_\infty}^2 \, d\mathrm{A} \qquad (1)$$

An estimate of free-stream velocity $\mathrm{U_\infty}$ at the WT location undisturbed by the WT itself, is necessary to close the model and interpolate into the manufacturer's thrust coefficient $\mathrm{C_T}$ curve (see Figure 2). An absolutely undisturbed velocity is obtained only through a precursor simulation, so that should be the standard against $\mathrm{U_\infty}$ estimates should be compared. The simplest way to estimate $\mathrm{U_\infty}$ is to consider the velocity some distance upstream of the WT (two diameters here) as an appropriate approximation. The proposed alternative is to use the velocity at the hub position (inside the actuator disk) as input to iterate Equation 3 with the $\mathrm{C_T}$ curve.

$$\begin{aligned} \mathrm{C_T} &= 4\mathrm{a}\,(1-\mathrm{a})\ , \\ \mathrm{a} &= 1 - \frac{\mathrm{U_{hub}}}{\mathrm{U_\infty}} \end{aligned} \qquad (2)$$

Knowing the applied force, actuator disk's mechanical power can be determined by integrating the force-velocity product over the disk's surface. Considering a given electrical-mechanical conversion efficiency $\eta_e$ of 97%, Equation 3 allows the model to estimate the WT's electical power.

$$d\mathrm{P} = \eta_e \, d\mathrm{F} \, \mathrm{U} \qquad (3)$$

## Results

As stated previously, the estimation of the free-stream velocity is key to predict the performance of a WT. Figure 1 shows that either estimation method proves to have high accuracy in the case of a WT in undisturbed flow. There are small deviations at low velocities from the $\mathrm{U_\infty}$ obtained from a precursor simulation, but the relative error falls with velocity.

When attempting to simulate the manufacturer's power curve, either prescribing the value for $\mathrm{U_\infty}$ used by the model, probing upstream or using the proposed momentum theory based iterative model seem to provide the same results (see Figure 2): reasonable agreement up to about 18 m/s, from whereon power estimation diverges significantly from the manufacturer's curve. This is believed to be due to a significant drop in aerodynamic efficiency of the rotor, result of some change in WT control strategy to limit electric power.

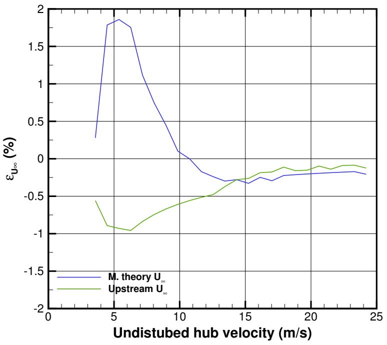

**Figure 1:** Deviation between free-stream velocity estimations and actual undisturbed velocity at WT hub over full wind speed range.

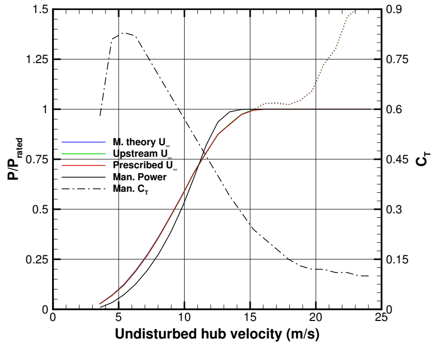

**Figure 2:** Power estimation over the WT's operating range, for different $\mathrm{U_\infty}$ estimation methods. Dotted lines show total modelled power if assuming absolute aerodynamic efficiency.

Upstream velocity probing proves limited when estimating free-stream velocity in the wake of another WT: comparison with the velocity undisturbed by the WT itself shows both an over-estimation of the velocity deficit and under-estimation of the wake width. The proposed method on the other hand shows good agreement on both terms.

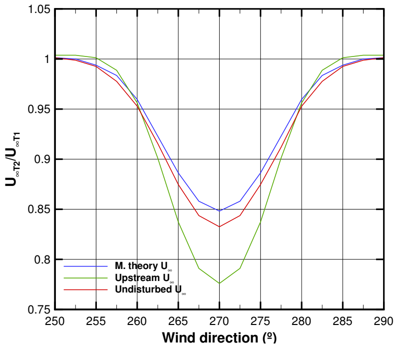

**Figure 3:** Free-stream velocity ratio for WT 5 diameters downstream of operating WT, for different $\mathrm{U_\infty}$ estimation methods.

## Conclusions

- For an isolated WT, $\mathrm{U_\infty}$ estimation is in good agreement with the undisturbed velocity at the hub position over the operating range;
- WT rotor's aerodynamic efficiency unaccounted for in manufacturer data, visible by detachment between modelled WT power and manufacturer's curve at the top end of the operating range; elsewhere reasonable agreement is achieved
- $\mathrm{U_\infty}$ estimation by the proposed method in good agreement with velocity at the WT position in disturbed flow;

## Acknowledgements

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## References

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